A Non-Gaussian Gust Model for Aircraft Response Analysis

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Abstract

NUMEROUS investigations have been performed on the dynamic response of aircraft to continuous atmospheric turbulence (gust) by assuming that the turbulence field can be modeled as one, or a number of individually Gaussian and stationary patches. However, the observed statistical properties of atmospheric turbulence encountered by aircraft, especially those with the highest severity and potentially most destructive, are in many cases non-Gaussian. For instance, Ref. 1 deals with the aeronautical effects of surface winds and gusts. The investigation revealed that turbulence can rarely be realistically represented by a Gaussian process. This paper describes a non-Gaussian model for continuous atmospheric turbulence based on zero-memory transformation of a Gaussian model. Single or multiple transformation parameters are used to control the severity and other statistical characteristics of the non-Gaussian process. The composite nature of gust patches of various intensity is accounted for through the accumulative statistics.

Methods to predict the aircraft response statistics to the non-Gaussian gust model are developed by approximating the response integral as its Riemann sum. Application of the probabilistic theory indicates that the statistical properties of response can be expressed in multifold integral forms and its moments can be expressed in explicit forms.

Contents

Experience has shown that the Gaussian gust model for stationary atmospheric turbulence within a patch leads to an underestimate of the occurrences of high-velocity gusts. The non-Gaussian gust model $X(t)$ demonstrated here is established by applying the hyperbolic sine function to a Gaussian process, $Y(t)$:

$$X(t) = g[Y(t)] = (2\sigma_X/\beta)\sinh(\gamma Y/\sigma_Y) \tag{1}$$

where σ_X and σ_Y are the standard deviations, β is a parameter, and

$$\gamma = [\tfrac{1}{2}\ln(\beta^2/2+1)]^{1/2} \tag{2}$$

Inversely,

$$Y(t) = f[X(t)] = (\sigma_Y/\gamma)\sinh^{-1}(\beta X/2\sigma_X) \tag{3}$$

The probability density function of $X_1 = X(t_1)$ is

$$p_{X_1}(x) = (2\pi)^{-1/2}(\beta/2\gamma\sigma_X)[1+(\beta x/2\sigma_X)^2]^{-1/2} \cdot \exp\{-(2\gamma^2)^{-1}[\sinh^{-1}(\beta x/2\sigma_X)]^2\} \tag{4}$$

The higher-order moments of the stationary random process $X(t)$ may be computed by

$$E[X^4] = \sigma_X^4(\beta^4/8+\beta^2+3) \tag{5a}$$

$$E[X^6] = 2\sigma_X^6 \frac{(\beta^2/2+1)^9 - 6(\beta^2/2+1)^4 + 15(\beta^2/2+1) - 10}{\beta^6} \tag{5b}$$

The expected rate of x-threshold crossings from below may be expressed as

$$N_X(x) = N_X(0)\exp\{-(2\gamma^2)^{-1}[\sinh^{-1}(\beta x/2\sigma_X)]^2\} \tag{6}$$

where

$$N_X(0) = (\sigma_{\dot X}/2\pi\sigma_X)(\beta/2\gamma)(1+\beta^2/4)^{-1/2} \tag{7}$$

Comparison of the statistical properties, including the probability density distribution, the exceedance statistics, and the kurtosis coefficient of the non-Gaussian gust model vs the parameter β are given in Fig. 1. From the plots, it can be seen that the patchy character of the non-Gaussian process (i.e., a higher probability for high gust velocities) is controlled by parameter β.

The composite gust model is made up of a number of patches of turbulent air of different severity, each stationary in character, encountered in random fashion. The random intensity distribution is represented by a probability function

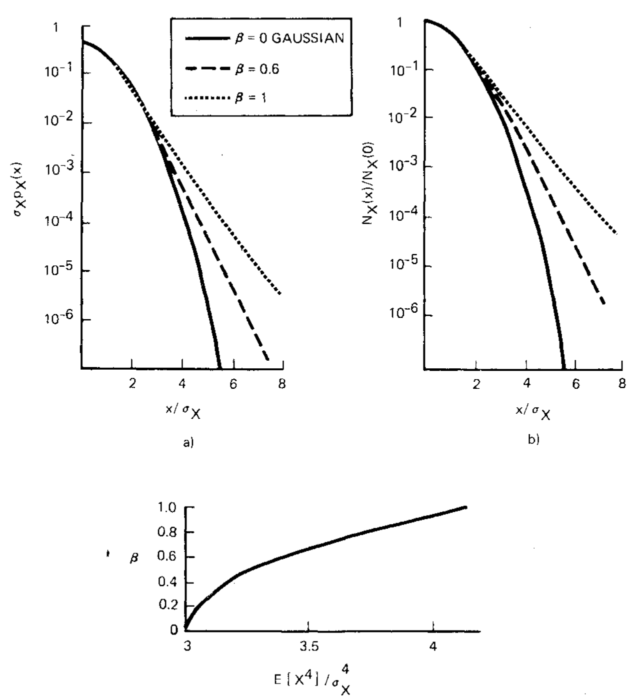

Fig. 1 Statistical properties of the non-Gaussian gust model: a) probability density, b) exceedance statistics, c) kurtosis coefficient.

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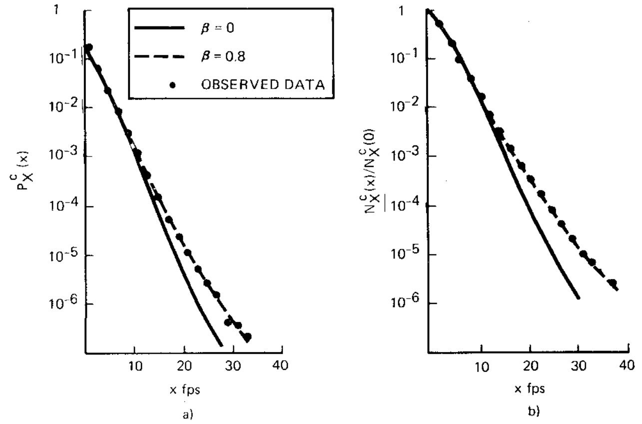

Fig. 2 Comparison of observed data to analytical predictions of composite non-Gaussian gust model: a) cumulative probability density, b) cumulative probability of exceedance.

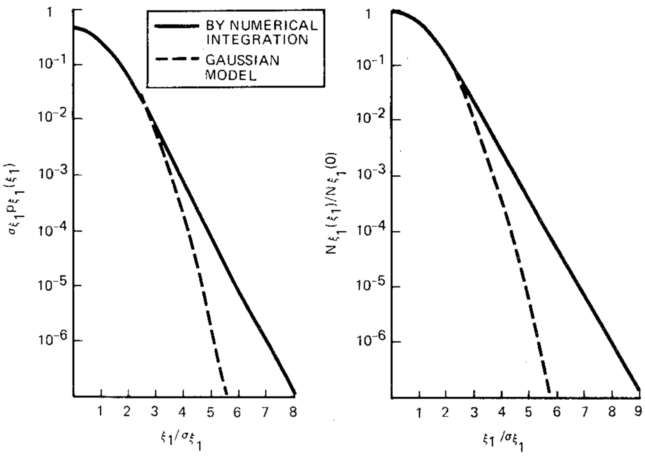

Fig. 3 Predicted response statistics for airplane plunging velocity to uniformly distributed non-Gaussian gusts.

$P_\sigma(\sigma_X)$. Assuming that the non-Gaussian transformation parameter β is constant for the various patches, the cumulative probability density function and the cumulative expected rate of level threshold crossings (from below) may be computed. By fitting the computed results to the corresponding curves of available experimental data, the value of β that yields the best agreement with the test data may be determined. For example, the empirical LO-LOCAT data for all contour flight conditions[2] are used to verify the non-Gaussian gust model. Using $\beta = 0.8$ for the analytical model, the resulting cumulative probability density distribution and normalized cumulative probability of exceedance for the vertical gust are plotted over experimental data. The results are shown in Fig. 2. Also plotted in the figure are the curves corresponding to the composite Gaussian model ($\beta = 0$).

The statistical prediction of aircraft response to the proposed non-Gaussian gust model is developed based on the probabilistic theory for stationary random process. Consider the response $Z(t)$ as an integral of the stationary non-Gaussian gust $X(t)$ and the impulse response function h, whose Fourier transformation $H(\omega)$ exists. The response may be approximated by

$$Z = \sum_{j=1}^{N} X_j h_j \tag{8}$$

By definition, the characteristic function of Z may be expressed in terms of the characteristic function of X by means of Eq. (8):

$$M_Z(\theta) = E[\exp(i\theta Z)] = M_{\{X\}}(\theta h_1,\ldots,\theta h_N) \tag{9}$$

Note that $M_{\{X\}}$ is the multifold Fourier transform of the non-Gaussian joint probability density function $p_{\{X\}}(x_1,\ldots,x_N)$. The inverse Fourier transform of $M_Z(\theta)$ gives the probability density function of Z:

$$p_Z(z) = h_1^{-1}\int\cdots\cdots_{(N-1)\,\text{fold}}\int_{-\infty}^{\infty} f'(\sigma)\, p_{\{Y\}}[f(\sigma), y_2,\ldots,y_N]$$

$$\cdot\, \mathrm{d}y_2 \ldots \mathrm{d}y_N \tag{10}$$

where

$$\sigma = \left[z - \sum_{j=2}^{N} h_j g(y_j)\right] \Big/ h_1 \tag{11}$$

A similar expression may be obtained for the expect rate of z-threshold crossings from below. Furthermore, the moments of the non-Gaussian gust response can be expressed in explicit forms (m = even):

$$E[Z^m] = \sigma_Z^m \left[\frac{\exp(\gamma^2/2)}{(\beta A)}\right]^m \sum_{j_1=1}^{N} \cdots \sum_{j_m=1}^{N} h_{j_1}\ldots h_{j_m}$$

$$\cdot \sum_{n_1=1}^{2} \cdots \sum_{n_m=1}^{2} a_{n_1}\ldots a_{n_m} \exp\left(\gamma^2 \sum_{i=1}^{m} \sum_{k>i}^{m} a_{n_i} a_{n_k} \rho_{j_i j_k}\right) \tag{12}$$

where

$$A = \sigma_Z/\sigma_X,\ \ a_1 = 1,\ \ a_2 = -1,\ \ \rho_{jk} = \rho_{kj} = \sigma_Y^{-2} E[Y_j Y_k]$$

The probability density function and the exceedance statistics of the response may be calculated by multifold numerical integration technique. The response of a Northrop F-5A airplane considering plunging and pitching modes subjected to uniformly distributed non-Gaussian gust is formulated, and the response statistics of the plunging mode are presented in Fig. 3 vs those for the Gaussian case. The corresponding kurtosis coefficient $E[Z^4]/\sigma_Z^4$ is 3.52. As expected, the numerical results indicate that the proposed model predicts higher probability of large responses than those yielded by the Gaussian model, which is important to structural designers.

References

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[2] Gunter, D. E., Jones, G. W., Jones, J. W., and Monson, K. R., "Low Altitude Atmospheric Turbulence, LO-LOCAT Phases I and II," Air Force Aeronautical Systems Div. ASD-TR-69-12, Feb. 1969.